Statistics 552 APPLIED STOCHASTIC PROCESSES
Spring 2017

Also see Course Announcement

Instructor: Marek Kimmel, Professor, DH 2102, x 5255, kimmel@rice.edu.
TA: TBA

Evaluation
30% homeworks, 15% midterm, 15% short quizzes, 40% final
Optional assignments and activity in class (asking questions) may provide extra credit

Honor Code
Homeworks can be discussed in groups. Final redaction has to be done personally. Verbatim copying of homework assignments is a violation.

Midterm and (in class) are under strict Honor Code rules and have to be completed without collaboration or help from others.

Short (5 min) quizzes will be given at random times in class and will concern material of any previous class or the current class. Please bring writing paper to class

Text:
Basic: Grimmett and Stirzaker "*Probability and Random Processes*" (3 ed.) Oxford, 2001 (reprinted last time 2009)
Course material Chapters 5, 6, 7
Background material (helpful) Chapters 1 - 4
Additional material on Brownian motion from Resnick "*Adventures in Stochastic Processes*"

Additional material on MCMC from source TBA

Lecture Notes and Study Guides: The class will be taught by performing mathematical derivations on the whiteboard with a sporadic use of projected slides.

Problem solving sessions: TA will call these at least once each two weeks.

Office hours: TA: To be announced.
One-to-one meetings with Prof. Kimmel, by appointment

Contents of the course (tentative; a partial overhaul is planned):

1. **Convergence of random variables**
2. **Martingales**
3. **Generating functions and their applications**
4. **Markov chains**
5. **Application of Markov chains in statistics: Markov Chain Monte Carlo**
6. **Renewal processes**
7. **Brownian Motion**

Course objectives and outcomes:

1. Develop understanding of the mathematical structures underlying time (and other) series influenced by chance.
2. Develop skills and appreciation for mathematically rigorous statements and proofs.
3. Provide foundations for applying stochastic processes theory in biology and other fields.
4. Practice the transition from heuristic understanding of the problems to placing them in the frame of the theory and developing solutions

Outcome: A personal toolbox of techniques to address problems in applications and advanced theory.

Students with disabilities are encouraged to attend: Regarding accomodations, please contact both the instructor and Disability Support Services at: http://dss.rice.edu/Content.aspx?ekfrm=2147483651 In particular: A student with a disability has a responsibility to contact DSS if any accommodations are required. If accommodations are needed, the student with a documented disability will receive an accommodation letter from DSS to take to each instructor. It is the student's responsibility to deliver these letters as soon as possible. Instructors expect to receive notification early in each semester; students who deliver their letters later may encounter difficulties. Good, early communication is vital to the accommodation process.

Changes: The syllabus is subject to change with reasonable advance notice by the instructor.

Stat 552-2016, Tentative contents

Material from lecture notes, book and additional materials, as listed below, problems similar to those in homework assignments

Abbreviations: pf = proof, thm = theorem, and so forth

Chapter 7. Convergence concepts

- Cauchy convergence: Definition, convergence implies Cauchy conv, def of completeness. Thm: Space of reals is complete (pf). Thm of Bolzano-Weierstrass (pf).
- 7.1. Convergence of functions (all).
- 7.2. Convergence types for rv. Def 1, Thm 3, Thm 4. Lemma 5 pf, Lemma 6 pf (counterexamples), Lemma 10a, b c (pf).

Conditional expectations and martingales (based on notes and Williams' book)

- *Refresher on expectations (based on Resnick's book slides):* Simple functions, approximation by simple functions, L-S integrals of simple functions, extension to non-negative measureable functions, properties of expectations, MCT no pf, Fatou lemmas no pf, BCT no pf, indefinite integrals, transformation Thm and densities no pf, Fubini Thm no pf.
- Intuitive introduction to conditional expectations (using simple functions)
- Definition of conditional expectations.
- Properties of conditional expectations (with proofs wherever handwritten proofs provided in the notes)
- Martingales, martingale transformations, upcrossings and proof of Doob's Theorem, with branching process and gambling examples

Brownian motion (…)

Chapter 5. Generating functions

- 5.1. Generating functions: Definition, de Moivre Thm pf using gf's, Ex. 4, 7 and 8, properties of power series, geometric series and differentiation example, Abel Thm, pgf's of constant, Bernoulli, binomial, Poisson, Thm 18 with pf, graphical representation of gf, representation as $E(s^X)$.
- 5.3. Simple Random Walk: Definition, being at 0 vs first return to 0, time to the first return, Thm for gf's related to SRW with pf, coroll about transience/recurrence, further results for SRW w/o pf: Thm 5, Coroll 6, reflection principle, Thm 7.
- 5.4 – 5.5 Branching Processes: Definition of the simple (Galton-Watson) bp, forward-time iteration and Thm 1 with pf, Ex 3, Lemma 2 with pf, subcritical, critical and supercritical bp, extinction, Thm 5 w pf, limit behavior in the supercritical case, age-dependent bp, backward balance equation, first moments by successive approximations, binary fission with exponential lifetimes leads to shifted geometric

- 5.7 – 5.9 Characteristic functions: Def 2 and Thm 3 with pf, Thm 4 without pf, Thm 5 and 6 with pf, Examples: Bernoulli, binomial, exponential, Cauchy via inverse Fourier transf (proof of inverse transform not required), Ex 5, 6, 7 and 8, convergence in distribution, Inversion and Continuity Theorems (no pf), uniqueness (with pf via inversion Thm), WLLN and CLT with pf.

Chapter 6. Markov Chains

- 6.1: Def 1 (Markov property and equivalent forms), Def 4, Thm 5, Def 6, Thm 7 (Chapman-Kolmogorov with pf and coroll), Lemma 8 (marginal distributions), Examples SRW, SBP, Ex 11 (Wright-Fisher and Moran models in genetics), Ex 12 (recurrent events vs MC), Ex 13.
- 6.2 Classification of States: Def 1 (recurrence and transience), first passage times, Thm 3 with pf, Coroll 4, other characterizations, null persistence, Defs 8, 10 and 11, Ex SBP.
- 6.3 Classification of Chains: Def. 1 (communication), decomposition into equivalence classes, Thm about intercommunication (pf of (b) required), Thm 4 (decomposition) with pf. Proposition: In a finite chain, all states cannot be transient (counterexample for an infinite chain).
- Classification in finite MC, finite gambler ruin example. Classification of states in SRW.
- Examples of stationary and limit distributions: spectral decomposition in the finite case.
- Prop, limit distribution is stationary
- 6.4. Stationarity, transience, recurrence, ergodocity: Def 1, Thm 3, Thm 6, Thm 10, Thm 17, Thm 21. Example of retaining boundary RW. Example of reversible mutations. Another infinite dim example. Proof of Thm 10.
- 6.5. Reversibility. Definitions. Thm 4 with examples.
- 6.9. Time-continuous MC. Def 1, Def 2, Thm 3 pf, Def 4, forward and backward equations. Birth process: condition of fairness pf, generator matrix for birth process, solution of forward equation using Laplace transform, Poisson process.
- 6.8. Def 1, Thm 2 (pf), Thm 10 (pf), Def 11, Eqs 12 and 13, Thm 14 (pf) and 16, Def 18, Lemma 20 (pf).
- 6.9. Cont. Interarrival times: Claim 13* pf, Claim 14* pf, Claim 20* pf, Ergodic Thm 21 pf sketch,
- 6.11. B-d processes. Stationarity, Ex 6, Ex 9, Thm 10, Equ 11, Cor 12,Thm 21.
- Rejection sampling and MCMC (from the notes and Gammerman's book): A/R algorithm and its application in Bayesian statistics, Metropolis and Metropolis-Hastings algorithms, Gibbs sampler, with proofs and examples as in the notes and HWs